Suppose that, last year, you purchased 150 shares of eight different stocks (for a total of 1200 shares). If you sell any shares, you have to pay a transaction cost of 1% of the amount transacted. In addition, you must pay a capital-gains tax at the rate of 30% on any capital gains at the time of the sale. You would like to sell enough shares of stock today to generate $10,000 to use as part of a down payment on a new home. You need to decide how many shares of which stocks to sell in order to generate $10,000, after taxes and transaction costs, while maximizing the estimated value of your stock portfolio next year. Let's formulate this as a linear optimization problem

Problem 1 - Formulating the Problem

How many decision variables should your model have?

Answer: 8, one for each stock.

What is the maximum value your decision variables can be?

Answer: 150

What is the minimum value your decision variables can be?

Answer: 0

Your objective is to maximize the estimated value of your stock portfolio next year. To do this, you should sum the estimated value of each stock next year. Suppose you sell x shares of your stock in Microsoft. What is the estimated value of your Microsoft stock next year?

Answer: (150-x)\*34.55

You need to make sure you get $10,000 in cash from selling your stocks, after taxes and transaction costs. How much would you get in cash, after taxes and transaction costs, if you sell 50 shares of your Intel stock?

Answer: 50\*23.67 - [0.01\*(50\*23.67) + 0.3(23.67-22.54)\*50] = $1154.72

Problem 2 - Analyzing the Solution

In the optimal solution, which stocks do you sell at least one share of? Select all that apply.

Answer: Yahoo!, Microsoft, Intel

What is the objective value of the optimal solution

Answer: $26773.66

How many shares of stock in total should you sell to make sure you have enough cash, according to the optimal solution?

Answer: 67.72329806 + 150 + 150 = 367.72329806

Problem 3 - Adjusting the Formulation

As an invester, you like having a portfolio of eight different stocks because it diversifies your investment. If one or two stocks do poorly this year, you won't worry as much because you have many other stocks. In the optimal solution for this problem, you sold all of your shares of some stocks, but you would like to keep at least half of the shares of each of your stocks.

Adjust the formulation so that you sell no more than 75 shares of each stock, and solve it again.

In the optimal solution, you sell at least one share of which of your stocks? Select all that apply.

Answer: All but Bank of America and JPMorgan Chase.

What is the objective value of the optimal solution now?

Answer: 26468.54116

Even though your investment is worth a bit less next year by diversifying, you expect this diversification to help you long term. However, you notice that you expect the Yahoo! stock to decrease in value next year. So, while you would like to sell no more than 75 shares of your other stocks, you would like to sell exactly 100 shares of your Yahoo! stock.

Adjust your formulation in Excel again, and re-solve to get the new optimal solution.

You now sell at least one share of how many different stocks?

Answer: 5

What is your estimated portfolio value next year?

Answer: $26507.53